

Transformer Model

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Abstract

This document describes a transformer model that is useful for the analysis of large scale electric power systems. It defines a balanced transformer representation primarily intended for use with analytical techniques that are based on the complex nodal admittance matrix. Particular attention is paid to the integration with "fast decoupled load flow" (FDLF) algorithms.

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I Transformer Equivalent Circuit

The equivalent circuit of a two winding transformer consists of an ideal transformer, a leakage impedance \mathbf{Z}_L , and a magnetizing impedance \mathbf{Z}_M . An ideal transformer is a lossless entity categorized by a complex voltage ratio \mathbf{a} , i.e.

$$\mathbf{a} = \frac{\mathbf{V}_p}{\mathbf{V}_s} \quad (1)$$

where

\mathbf{V}_p is the transformer's primary voltage.

\mathbf{V}_s is the transformer's secondary voltage.

Since the ideal transformer has no losses, it must be true that

$$\mathbf{S}_p = \mathbf{S}_s \quad (2)$$

or

$$\mathbf{V}_p \mathbf{I}_p^* = \mathbf{V}_s \mathbf{I}_s^* \quad (3)$$

Solving for the secondary current

$$\mathbf{I}_s^* = \left(\frac{\mathbf{V}_p}{\mathbf{V}_s} \right) \mathbf{I}_p^* \quad (4)$$

$$\mathbf{I}_s = \left(\frac{\mathbf{V}_p}{\mathbf{V}_s} \right)^* \mathbf{I}_p \quad (5)$$

Substituting [Equation 1](#) reveals the relationship between the primary and secondary current of an ideal transformer

$$\mathbf{I}_s = \mathbf{a}^* \mathbf{I}_p \quad (6)$$

where

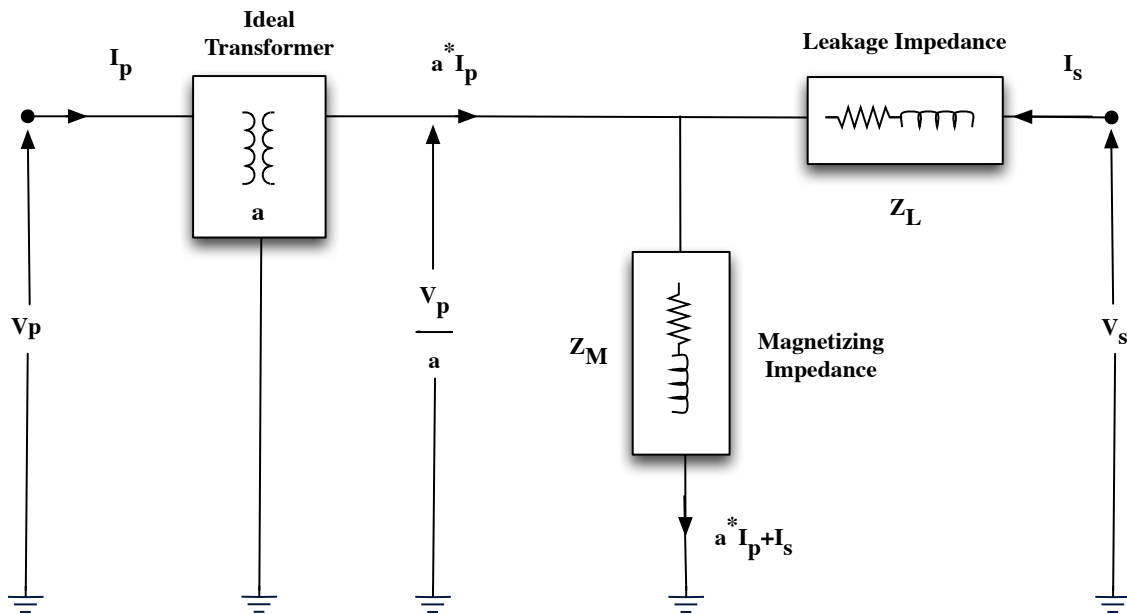
\mathbf{I}_p is current into the transformer's primary.

\mathbf{I}_s is current out of the transformer's secondary.

\mathbf{a} is the transformer's voltage transformation ratio.

A transformer's magnetizing impedance is a shunt element associated with its excitation current, i.e. the "no load" current in the primary windings. The real component of the magnetizing impedance reflects the core losses of the transformer. Its reactive

Figure 1: Transformer Equivalent Circuit Diagram



component describes the *mmf* required to overcome the magnetic reluctance of the core.

A transformer's leakage impedance is a series element that reflects imperfections in the windings. The real component of the leakage impedance is the resistance of the windings. Its reactive component is due to leakage flux, i.e. flux in the magnetic circuit that does not contribute to coupling the windings.

Figure 1 depicts a transformer's equivalent circuit with impedances referred to the secondary.

2 Transformer Parameters

The objective of this section is review commonly published transformer data and examine techniques for massaging raw transformer data into the form required by the parameters of analysis algorithms that are based on the complex nodal admittance matrix \mathbf{Y}_{bus} .

If magnetizing impedance is neglected, the steady state characteristics of a transformer are captured by its leakage impedance, voltage transformation properties (i.e. transformation ratio and phase shift). Tap-changing-under-load (TCUL) devices are also characterized by their voltage regulation capabilities.

2.1 Leakage Impedance

A transformer's leakage impedance is determined empirically through a short-circuit test. Leakage impedances provided by equipment manufacturers are usually expressed in percent using the device's rated power as the base. To convert the impedance from percent to per unit, just divide it by a hundred.

$$\mathbf{Z}_{\text{pu}} = \frac{\mathbf{Z}_{\%}}{100} \quad (7)$$

To express the impedance in per unit on the system base rather than the transformer base

$$\mathbf{Z}'_{\text{pu}} = \mathbf{Z}_{\text{pu}} \frac{\mathbf{S}'_{\text{base}} \mathbf{V}_{\text{base}}}{\mathbf{S}_{\text{base}} \mathbf{V}'_{\text{base}}} \quad (8)$$

where

\mathbf{S}_{base} is the transformer power base.

\mathbf{V}_{base} is the transformer voltage base.

$\mathbf{S}'_{\text{base}}$ is the system power base.

$\mathbf{V}'_{\text{base}}$ is the system voltage base.

When the transformer voltage base coincides with the system voltage base, this equation simplifies to

$$\mathbf{Z}'_{\text{pu}} = \mathbf{Z}_{\text{pu}} \left(\frac{\mathbf{S}'_{\text{base}}}{\mathbf{S}_{\text{base}}} \right) \quad (9)$$

The transformer's impedance may also be expressed in ohms

$$\mathbf{Z}_{\Omega} = \mathbf{Z}_{\text{pu}} \left(\frac{\mathbf{V}_{\text{rated}}^2}{\mathbf{S}_{\text{rated}}} \right) c \quad (10)$$

where

$\mathbf{V}_{\text{rated}}$ is the transformer's rated voltage. The computed impedance is referred to the transformer's primary or secondary by the appropriate choice of $\mathbf{V}_{\text{rated}}$.

$\mathbf{S}_{\text{rated}}$ is the transformer's power rating.

c is a constant that keeps the units straight. The value of c depends on how the transformer's power and voltage ratings are specified. Variations in c over a wide range of unit systems are found in [Table 1](#).

Table 1: Per Unit to Ohm Conversion Constants

Voltage Unit	Power Unit	Constant
V_{ln}	$VA_{1\phi}$	1
V_{ln}	$VA_{3\phi}$	3
V_{ln}	$kVA_{1\phi}$	1/1,000
V_{ln}	$kVA_{3\phi}$	3/1,000
V_{ll}	$VA_{1\phi}$	1/3
V_{ll}	$VA_{3\phi}$	1
V_{ll}	$kVA_{1\phi}$	1/3,000
V_{ll}	$kVA_{3\phi}$	1/1,000
kV_{ln}	$kVA_{1\phi}$	1,000
kV_{ln}	$kVA_{3\phi}$	3,000
kV_{ln}	$MVA_{1\phi}$	1
kV_{ln}	$MVA_{3\phi}$	3
kV_{ll}	$kVA_{1\phi}$	1,000/3
kV_{ll}	$kVA_{3\phi}$	1,000
kV_{ll}	$MVA_{1\phi}$	1/3
kV_{ll}	$MVA_{3\phi}$	1

2.2 Voltage Transformation

A transformer's voltage ratio is expressed in polar form as $ae^{j\delta}$, where a is the magnitude of the transformation and δ is its phase shift.

2.2.1 Transformation Ratio

Each transformer is categorized by a nominal operating point, i.e. its nameplate primary and secondary voltage. The nominal magnitude of the voltage ratio is determined by examining the real part of Equation 1 at nominal operating conditions.

$$a_{nom} = \frac{V_p}{V_s} \quad (11)$$

where

V_p is the nominal primary voltage.

V_s is the nominal secondary voltage.

When voltages are expressed in per unit, the nominal value of a is always one.

2.2.2 Phase Shift

In the general case, an arbitrary angular shift δ may be introduced by a multiphase transformer bank. However, omitting phase shifters (angle regulating transformers) from the system model, reduces the angle shifts across balanced transformer configurations to effects introduced by the connections of the windings. [Table 2](#) describes the phase shift associated with common transformer connections.

Table 2: Transformer Phase Shifts

Winding		Phase Shift
Primary	Secondary	
Wye	Wye	0°
Wye	Delta	-30°
Delta	Wye	30°
Delta	Delta	0°

When phase shifters are ignored, the user does not have to specify δ explicitly. It may be inferred from [Table 2](#).

2.3 Voltage Regulation

Most power transformers permit you to tap into the windings at a variety of different positions. Some transformers are configured with voltage sensors and motors which permit them to change tap settings automatically in response to voltage variations on the power system. These transformers are referred to as tap-changing-under-load (TCUL) devices. A TCUL device that controls voltage but does not transform large amounts of power is referred to as a regulating transformer. Regulating transformers that control voltage magnitude on distribution systems are usually called voltage regulators or simply regulators.

TCUL devices that regulate voltage magnitudes are categorized by a tap range and a tap increment. The tap range defines the limits of the device's regulating ability. The tap increment defines the resolution of the device. The tap changing mechanism of many transformers is categorized by a regulation range and a step count, e.g. ± 10 percent, 32 steps. The tap increment is computed from this data as follows.

$$T_{inc} = \frac{T_{max} - T_{min}}{n_{steps}} \quad (12)$$

where

T_{inc} is the transformer's tap increment.

T_{max} is maximum increase in voltage provided by the transformer (sometimes called boost). If the transformer can not boost the voltage, T_{max} is zero.

T_{min} is maximum decrease in voltage provided by the transformer (sometimes called buck). If the transformer can not decrease the voltage, T_{min} is zero.

n_{steps} is the number of tap steps.

Table 3 catalogs the tap range and increment of common load tap-changers.

Table 3: Common Transformer Tap Parameters

Regulation Range	n_{steps}	T_{max}	T_{min}	T_{inc}
$\pm 10\%$	32	10.0	-10.0	0.62500
$\pm 10\%$	16	10.0	-10.0	1.25000
$\pm 10\%$	8	10.0	-10.0	2.50000
+10%	32	10.0	0.0	0.31250
+10%	16	10.0	0.0	0.62500
+10%	8	10.0	0.0	1.25000
$\pm 7.5\%$	32	7.5	-7.5	0.46875
$\pm 7.5\%$	16	7.5	-7.5	0.93750
$\pm 7.5\%$	8	7.5	-7.5	1.87500
+7.5%	32	7.5	0.0	0.23437
+7.5%	16	7.5	0.0	0.46875
+7.5%	8	7.5	0.0	0.93750
$\pm 5\%$	32	5.0	-5.0	0.31250
$\pm 5\%$	16	5.0	-5.0	0.62500
$\pm 5\%$	8	5.0	-5.0	1.25000
+5%	32	10.0	0.0	0.15625
+5%	16	10.0	0.0	0.31250
+5%	8	10.0	0.0	0.62500

3 Transformer Admittance Model

For analysis purposes, each transformer is described by the admittances of a general two-port network. When the current equations of Figure 1 are written as follows, the transformer admittances correspond to the coefficient matrix.

$$\mathbf{I}_p = \mathbf{Y}_{pp} \mathbf{V}_p + \mathbf{Y}_{ps} \mathbf{V}_s \quad (13)$$

$$\mathbf{I}_s = \mathbf{Y}_{sp} \mathbf{V}_p + \mathbf{Y}_{ss} \mathbf{V}_s \quad (14)$$

where

\mathbf{Y}_{pp} is the driving point admittance of the primary.

\mathbf{Y}_{ss} is the driving point admittance of the secondary.

\mathbf{Y}_{ps} is the transfer admittance from the primary to the secondary.

\mathbf{Y}_{sp} is the transfer admittance from the secondary to the primary.

Noting that

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L}$$

and

$$\mathbf{Y}_M = \frac{1}{\mathbf{Z}_M}$$

It is apparent from inspection of [Figure 1](#) that

$$\mathbf{a}^* \mathbf{I}_p = \left(\frac{\mathbf{V}_p}{\mathbf{a}} - \mathbf{V}_s \right) \mathbf{Y}_L + \mathbf{Y}_M \frac{\mathbf{V}_p}{\mathbf{a}}$$

and

$$\mathbf{I}_s = \left(\mathbf{V}_s - \frac{\mathbf{V}_p}{\mathbf{a}} \right) \mathbf{Y}_L$$

Converting these equations to the desired form

$$\mathbf{I}_p = \left(\frac{1}{\mathbf{a}\mathbf{a}^*} \right) (\mathbf{Y}_L + \mathbf{Y}_M) \mathbf{V}_p - \left(\frac{\mathbf{Y}_L}{\mathbf{a}^*} \right) \mathbf{V}_s \quad (15)$$

$$\mathbf{I}_s = - \left(\frac{\mathbf{Y}_L}{\mathbf{a}^*} \right) \mathbf{V}_p + \mathbf{Y}_L \mathbf{V}_s \quad (16)$$

Recalling that $\mathbf{a}\mathbf{a}^* = a^2$ and equating the coefficients in [Equation 13](#) and [Equation 14](#) with their counterparts in [Equation 15](#) and [Equation 16](#), yields the transformer's admittances:

$$\mathbf{Y}_{pp} = \frac{\mathbf{Y}_L + \mathbf{Y}_M}{a^2} \quad (17)$$

$$\mathbf{Y}_{ss} = \mathbf{Y}_L \quad (18)$$

$$\mathbf{Y}_{ps} = -\frac{\mathbf{Y}_L}{\mathbf{a}^*} \quad (19)$$

$$\mathbf{Y}_{sp} = -\frac{\mathbf{Y}_L}{\mathbf{a}} \quad (20)$$

In [Equation 17](#) the real number a is the absolute value of the complex voltage ratio. When the magnetizing impedance is quite large, \mathbf{Y}_M approaches zero and [Equation 17](#) simplifies to

$$\mathbf{Y}_{pp} = \frac{\mathbf{Y}_L}{a^2}$$

The following sections examine data and computations required to incorporate this model into balanced power system analyses.

4 Computing Admittances

This section examines Equation 17 through Equation 20 with an eye for streamlining their computation. The resulting computational sequence explicitly decomposes the complex equations into real operations.

A transformer's leakage admittance \mathbf{Y}_L is derived from the leakage impedance \mathbf{Z}_L in the usual manner. Expressing the leakage impedance in rectangular form

$$\mathbf{Z}_L = r_L + jx_L \quad (21)$$

then

$$\mathbf{Y}_L = \frac{1}{\mathbf{Z}_L} = \frac{1}{r_L + jx_L} = g_L + jb_L \quad (22)$$

where

$$g_L = \frac{r_L}{r_L^2 + x_L^2} \quad (23)$$

$$b_L = -\frac{x_L}{r_L^2 + x_L^2} \quad (24)$$

For the sake of convenience, define the tap ratio as

$$\mathbf{t} = \frac{1}{\mathbf{a}} = te^{j-\delta} \quad (25)$$

4.1 Self Admittances

In the context of modeling and analysis of electrical networks, the self admittances of a transformer can be thought of as the device's impact on the diagonal elements of the nodal admittance matrix \mathbf{Y}_{bus} .

4.1.1 Primary Admittance

Considering the self admittance of the transformer's primary, Equation 17 states

$$\mathbf{Y}_{\text{pp}} = \frac{\mathbf{Y}_L + \mathbf{Y}_M}{a^2} \quad (26)$$

Expressing the complex equation in rectangular form and making the substitution defined in Equation 25

$$\mathbf{Y}_{\text{pp}} = t^2(g_L + g_M) + jt^2(b_L + b_M) \quad (27)$$

When the magnetizing admittance is ignored, Equation 27 reduces to

$$\mathbf{Y}_{\text{pp}} = t^2g_L + jt^2b_L \quad (28)$$

4.1.2 Secondary Admittance

Equation 18 defines the self admittance of the transformer's secondary as

$$\mathbf{Y}_{ss} = \mathbf{Y}_L \quad (29)$$

which is expressed in rectangular coordinates as

$$\mathbf{Y}_{ss} = g_L + jb_L \quad (30)$$

4.2 Transfer Admittances

In the context of modeling and analysis of electrical networks, the transfer admittances of a transformer can be thought of as the device's impact on the off-diagonal elements of of the nodal admittance matrix \mathbf{Y}_{bus} .

4.2.1 Primary to Secondary

Equation 19 defines the transfer admittance from the transformer's primary to its secondary as

$$\mathbf{Y}_{ps} = -\frac{\mathbf{Y}_L}{\mathbf{a}^*} \quad (31)$$

Substituting Equation 25 and resolving the tap ratio into polar form

$$\mathbf{Y}_{ps} = -\mathbf{t}^* \mathbf{Y}_L = -te^{j\delta} \mathbf{Y}_L$$

Expressing the tap ratio and leakage admittance in rectangular form

$$\mathbf{Y}_{ps} = -t(\cos \delta + j \sin \delta)(g_L + jb_L)$$

Carrying out the multiplications then collecting real and imaginary terms

$$\mathbf{Y}_{ps} = -t((-b_L \sin \delta + g_L \cos \delta) + j(g_L \sin \delta + b_L \cos \delta))$$

Therefore,

$$\mathbf{Y}_{ps} = -t(g_{ps} + jb_{ps}) \quad (32)$$

where

$$g_{ps} = -b_L \sin \delta + g_L \cos \delta \quad (33)$$

and

$$b_{ps} = g_L \sin \delta + b_L \cos \delta \quad (34)$$

4.2.2 Secondary to Primary

Equation 20 defines the transfer admittance from the transformer's secondary to its primary as

$$\mathbf{Y}_{sp} = -\frac{\mathbf{Y}_L}{\mathbf{a}} \quad (35)$$

Substituting Equation 25 and resolving the tap ratio into polar form

$$\mathbf{Y}_{sp} = -\mathbf{t}^* \mathbf{Y}_L = -t e^{j\delta} \mathbf{Y}_L$$

Expressing the tap ratio and leakage admittance in rectangular form

$$\mathbf{Y}_{sp} = -t(\cos(-\delta) + j\sin(-\delta))(g_L + jb_L)$$

Invoking the trigonometric identities $\cos(-\delta) = \cos \delta$ and $\sin(-\delta) = -\sin \delta$ the equation becomes

$$\mathbf{Y}_{sp} = -t(\cos \delta - j\sin \delta)(g_L + jb_L)$$

Carrying out the multiplications then collecting real and imaginary terms

$$\mathbf{Y}_{sp} = -t((b_L \sin \delta + g_L \cos \delta) + j(-g_L \sin \delta + b_L \cos \delta))$$

Therefore,

$$\mathbf{Y}_{sp} = -t(g_{sp} + jb_{sp}) \quad (36)$$

where

$$g_{sp} = b_L \sin \delta + g_L \cos \delta \quad (37)$$

and

$$b_{sp} = -g_L \sin \delta + b_L \cos \delta \quad (38)$$

4.2.3 Observations on Computational Sequence

The preceding results suggest the computational sequence is not too crucial. The values $\sin \delta$ and $\cos \delta$ appearing in Equation 32 and Equation 36 should only be computed once. Marginal benefits may also accrue from a single computation of the terms $g_L \sin \delta$, $g_L \cos \delta$, $b_L \sin \delta$, $b_L \cos \delta$ and t^2 .

5 TCUL Devices

A voltage regulating transformer changes its tap ratio to maintain a constant voltage magnitude at its regulation point. The normalized change in voltage (denoted by Δv) required to bring the regulation point to its designated value is just

$$\Delta v = \frac{V_k^{sp} - V_k}{V_k} \quad (39)$$

where

k is the regulated vertex.

V_k^{sp} is the specified (desired) voltage at vertex k , the regulation point.

V_k is the actual voltage at vertex k .

Assuming that the normalized change in voltage at the regulation point is proportional to the normalized change in voltage at the transformer's secondary, the secondary voltage must change as follows

$$V_s^{new} = V_s^{old} + \alpha \Delta v V_s^{old} \quad (40)$$

where α is the constant of proportionality.

5.1 Computing Tap Adjustments

Recalling that the secondary voltage of a transformer is a function of the tap ratio

$$V_s = tV_p$$

implies that

$$V_s^{new} = t^{new}V_p$$

and

$$V_s^{old} = t^{old}V_p$$

Substituting these values into [Equation 40](#) yields

$$t^{new}V_p = t^{old}V_p + \alpha \Delta v t^{old}V_p$$

which simplifies to

$$t^{new} = t^{old} + \alpha \Delta v t^{old} \quad (41)$$

To obtain a computational formula, substitute [Equation 39](#)

$$t^{new} = t^{old} + \alpha t^{old} \frac{V_k^{sp} - V_k}{V_k}$$

and simplify as follows

$$t^{new} = t^{old} + \alpha t^{old} \left(\frac{V_k^{sp}}{V_k} - 1 \right) = t^{old} + \alpha t^{old} \frac{V_k^{sp}}{V_k} - \alpha t^{old}$$

or

$$t^{new} = t^{old} \left(1 - \alpha + \alpha \frac{V_k^{sp}}{V_k} \right) \quad (42)$$

Normally, α is assumed to be one and [Equation 42](#) reduces to

$$t^{new} = t^{old} \left(\frac{V_k^{sp}}{V_k} \right) \quad (43)$$

Computation of α , the sensitivity of the regulated voltage to transformer tap changes, is discussed in [Section 5.3](#).

5.2 Error Feedback Formulation

Stott and Alsac ([1974](#)) and Chan and Brandwajn ([1986](#)) describe transformer tap adjustments as an error feedback formula

$$a^{new} - a^{old} = \alpha \left(\frac{V_k^{sp}}{V_k} \right) \quad (44)$$

where all quantities are expressed in per unit. The per unit voltage transformation ratio a^{pu} is computed as follows

$$a^{pu} = \frac{V_p^{pu}}{V_s^{pu}} = \frac{\frac{V_p}{V_p^{nom}}}{\frac{V_s}{V_s^{nom}}}$$

which simplifies to

$$a^{pu} = \frac{V_s^{nom} \frac{V_p}{V_p^{nom}}}{V_s} = \left(\frac{V_p}{V_s} \right) \left(\frac{V_s^{nom}}{V_p^{nom}} \right) = \frac{a}{a^{nom}}$$

where a^{nom} is the nominal voltage transformation ratio. Therefore, [Equation 44](#) is equivalent to

$$\frac{a^{new}}{a^{nom}} - \frac{a^{old}}{a^{nom}} = \alpha \frac{V_k - V_k^{sp}}{V_k^{base}}$$

or

$$a^{new} - a^{old} = a^{nom} \alpha \frac{V_k - V_k^{sp}}{V_k^{base}}$$

in physical quantities. Solving for the updated transformation ratio yields

$$a^{new} = a^{old} + a^{nom} \alpha \frac{V_k - V_k^{sp}}{V_k^{base}} \quad (45)$$

Computation of α , the sensitivity of the regulated voltage to changes in transformation ratio, is discussed in [Section 5.3](#).

5.3 Controlled Bus Sensitivity

The proportionality constant α (referenced in [Equation 45](#) and [Equation 42](#)) represents the sensitivity of the controlled voltage to tap changes at the regulating transformer. It is commonly assumed that α is one (the controlled voltage is perfectly sensitive to tap changes). An alternate assumption is that

$$\alpha = \mathbf{M}_k^T (\mathbf{B}'')^{-1} \mathbf{N} \quad (46)$$

where

\mathbf{M}_k^T is a row vector with 1 in the k^{th} position.

\mathbf{N} is a column vector with $-b_{ps}t$ in position p and $b_{ps}t$ in position s .

Carrying out these matrix operations yields

$$\alpha = -b_{ps}t b_{kp}''^{-1} + b_{ps}t b_{ks}''^{-1} \quad (47)$$

where

b_{ij} is an element from the nodal susceptance matrix \mathbf{B}_{bus} .

$b_{ij}''^{-1}$ is an element from the inverse of \mathbf{B}'' as defined by Stott and Alsac ([1974](#)).

t is magnitude of the transformer's tap ratio.

Chan and Brandwajn ([1986](#)) suggest that sensitivities computed from [Equation 47](#) are useful for coordinating adjustments to a bus that is controlled by several transformers.

5.4 Physical Constraints on Tap Settings

The preceding sections treat a transformer's tap setting as a continuous quantity. However, tap settings are discrete. There are three ways to handle discrete taps:

- Modify equations [Equation 42](#) and [Equation 45](#) so that the tap setting is computed discretely.
- Compute tap settings from [Equation 42](#) or [Equation 45](#). Round the tap setting to the nearest discrete value each time the tap changes.
- Compute tap settings from [Equation 42](#) or [Equation 45](#). Round the tap settings after the load flow has converged. Fix the transformer taps. Continue until the solution converges with the discrete fixed tap transformers.

As was discussed in [Section 2.3](#), the tap changing mechanism is limited in its regulating abilities. Each TCUL device is subject to tap limits, i.e. maximum and minimum physically realizable tap settings. The implementation of [Equation 42](#), [Equation 43](#), or [Equation 45](#) should reflect this fact. For example, [Equation 43](#) should be implemented as

$$t^{new} = \begin{cases} t_{min} & \text{if } t^{new} < t_{min} \\ t^{old} \left(\frac{V_k^{sp}}{V_k} \right) & \text{if } t_{min} \leq t^{new} \leq t_{max} \\ t_{max} & \text{if } t^{new} > t_{max} \end{cases} \quad (48)$$

6 Transformers in the Nodal Admittance Matrix

The complex nodal admittance matrix is referred to as \mathbf{Y}_{bus} . In the current formulation, the admittances of \mathbf{Y}_{bus} are stored in rectangular form. The real part of \mathbf{Y}_{bus} (the conductance matrix) is called G_{bus} . The imaginary part of \mathbf{Y}_{bus} (the susceptance matrix) is called B_{bus} . \mathbf{Y}_{bus} is formed according to two simple rules.

1. The diagonal terms of \mathbf{Y}_{bus} (i.e. \mathbf{y}_{ii}) are the driving point admittances of the network. They are the algebraic sum of all admittances incident upon vertex i . These values include both the self admittances of incident edges (with respect to vertex i) and shunts to ground at the vertex itself.
2. The off-diagonal terms of \mathbf{Y}_{bus} (i.e. \mathbf{y}_{ij} where $i \neq j$) are the transfer admittances of the edge connecting vertices i and j .

Assuming that each transformer is modeled as two vertices (representing its primary and secondary terminals) and a connecting edge in the network graph, the procedure for incorporating a transformer into the nodal admittance matrix is as follows.

1. Compute the admittance matrix of the transformer (i.e. \mathbf{Y}_{pp} , \mathbf{Y}_{ss} , \mathbf{Y}_{ps} , and \mathbf{Y}_{sp}) at the nominal tap setting. See [Section 4](#) for details.

2. Include the transformer's primary admittance \mathbf{Y}_{pp} as its contribution to the self admittance of the primary vertex p . That is, add \mathbf{Y}_{pp} to the corresponding diagonal element of the nodal admittance matrix \mathbf{y}_{pp} .
3. Include the transformer's secondary admittance \mathbf{Y}_{ss} as its contribution to the self admittance of the secondary vertex s . That is, add \mathbf{Y}_{ss} to the corresponding diagonal element of the nodal admittance matrix \mathbf{y}_{ss} .
4. Use \mathbf{Y}_{ps} as the transfer admittance of the edge from the primary vertex p to the secondary vertex s (i.e. set the corresponding off-diagonal element of the nodal admittance matrix \mathbf{y}_{ps} to \mathbf{Y}_{ps}).
5. Use \mathbf{Y}_{sp} as the transfer admittance of the edge from the secondary vertex s to the primary vertex p (i.e. set the corresponding off-diagonal element of the nodal admittance matrix \mathbf{y}_{sp} to \mathbf{Y}_{sp}).

Note: Examining Equation 19 and Equation 20, you can see that asymmetries introduced into \mathbf{Y}_{bus} by a transformer are due to the fact that \mathbf{a}^* is used to compute \mathbf{Y}_{ps} and \mathbf{a} is used to compute \mathbf{Y}_{sp} . When $\mathbf{a} = \mathbf{a}^*$, \mathbf{Y}_{bus} is symmetric with respect to transformers. This condition is only true when \mathbf{a} is real. In this situation, δ is zero and there is no phase shift across the transformer.

7 The Nodal Admittance Matrix during TCUL

Tap-changing-under-load (TCUL) has an impact on power flow studies since it changes the voltage transformation ratio thus changing the impedance of a transformer. These impedance adjustments must be reflected in the nodal admittance matrix \mathbf{Y}_{bus} . The following discussion examines techniques for updating \mathbf{Y}_{bus} when a transformer changes tap settings.

Assume that the following information is required by any procedure that updates \mathbf{Y}_{bus} following a tap change:

- \mathbf{Y}_{bus} itself,
- \mathbf{Y}_L the transformer's leakage admittance, and
- \mathbf{t}_{new} the transformer's tap ratio after the tap change.

The transfer admittances (off-diagonal elements of \mathbf{Y}_{bus}) can be maintained with this base of information. However, maintaining the self admittances is more complicated. There are two strategies for updating the diagonal of \mathbf{Y}_{bus} .

- Rebuild the entry from scratch, or
- Back out the old transformer admittances, then add in the new values.

The reconstruction strategy is ruled out as computationally inefficient without extensive examination. Computational requirements of rebuilding an entry include:

- Compute the new admittances of the TCUL device.
- Access the incidence list of the primary vertex p .
- Look up shunt elements (e.g. impedance loads) at the primary vertex p .
- Recompute the self admittances of any transformers adjacent to the TCUL device (incident upon its primary vertex).

The direct update strategy is examined more carefully. Its basic computational complexity stems from backing the old value of the transformer's primary admittance \mathbf{Y}_{pp} out of \mathbf{y}_{pp} (the self admittance of vertex p). There are two ways to proceed:

1. Remember \mathbf{Y}_{pp} so that it can be backed out directly, or
2. Recompute \mathbf{Y}_{pp} , then back it out of \mathbf{y}_{pp} . From Equation 27 or Equation 28, it is seen that the square of t_{old} (the magnitude of the original tap ratio) is needed to recompute \mathbf{Y}_{pp} .

The \mathbf{Y}_{bus} maintenance algorithm based on a direct update approach follows.

1. Determine the transformer's original primary admittance \mathbf{Y}_{pp} , subtract it from the corresponding diagonal term of the nodal admittance matrix \mathbf{y}_{pp} .
2. Compute the new primary admittance \mathbf{Y}_{pp} using Equation 27 or Equation 28, add it to the corresponding diagonal term of the nodal admittance matrix \mathbf{y}_{pp} .
3. Compute the new transfer admittance \mathbf{Y}_{ps} from Equation 32, set the corresponding off-diagonal element of the admittance matrix \mathbf{y}_{ps} to this value.
4. Compute the new transfer admittance \mathbf{Y}_{sp} using Equation 36, set the corresponding off-diagonal element of the admittance matrix \mathbf{y}_{sp} to this value.

Examining Equation 30, you should observe that the transformer's secondary admittance \mathbf{Y}_{ss} is not a function of the tap setting. Therefore, it does not have to be updated when the tap changes.

When δ does not change during the load flow (i.e. no phase angle regulators), the terms $g_L \sin \delta$, $g_L \cos \delta$, $b_L \sin \delta$, $b_L \cos \delta$ in Equation 32 and Equation 36 remain constant throughout the load flow solution. If these terms are precomputed and stored, a considerable reduction in floating point arithmetic results. Obviously, you pay a price in storage. Table 4 summarizes the time and space trade-offs associated with recomputation vs storage of these products when updating the diagonal of \mathbf{Y}_{bus} and computing the transfer admittances \mathbf{Y}_{ps} and \mathbf{Y}_{sp} in the absence of phase angle regulators.

8 Integrating TCUL into FDLF

A fast decoupled load flow (FDLF) iteration consists of four basic steps:

1. Compute real power mismatches.

Table 4: \mathbf{Y}_{bus} Maintenance Costs

Operation	FLOPS	Storage	
		Required	Assumed
Update \mathbf{Y}_{bus} Diagonals			
Recompute	$19n_t + 4n_t$	$n_t n_{\text{float}}$	$4n_t$
Store	$19n_t + 2n_t$	$2n_t n_{\text{float}}$	$8n_t$
Update \mathbf{Y}_{ps} and \mathbf{Y}_{sp}			
Recompute	$19n_t$	$n_t n_{\text{float}}$	$4n_t$
Store	$9n_t$	$4n_t n_{\text{float}}$	$16n_t$

n_t is the number of TCUL devices.

n_{float} is the size of a floating point number in bytes.

19 FLOPS computes \mathbf{Y}_{pp} , \mathbf{Y}_{ps} , and \mathbf{Y}_{sp} . Updates \mathbf{y}_{pp} .

9 FLOPS computes \mathbf{Y}_{pp} , \mathbf{Y}_{ps} , \mathbf{Y}_{sp} . Updates \mathbf{y}_{pp} w/constant angle.

2 FLOPS moves \mathbf{Y}_{pp} into or out of \mathbf{y}_{pp} .

2 FLOPS recomputes \mathbf{Y}_{pp} from t_{old}^2 .

2. Update the voltage angles.
3. Compute reactive power mismatches.
4. Update the voltage magnitudes.

The tap adjustment algorithm is applied after the voltage magnitude update.

Chan and Brandwajn (1986) suggest that instability is introduced into the solution by adjusting TCUL devices before the load flow is moderately converged. Operationally, moderate convergence is defined by starting criterion for tap adjustments. The criterion is either:

- A minimum iteration count (no transformer taps are changed before the designated iteration) or
- A maximum bus mismatch (no transformer taps are changed unless the maximum reactive bus mismatch is less than the criterion).

Chan and Brandwajn (1986) also suggest that an auxiliary solution be performed after the tap update. The tap adjustment creates an incremental reactive mismatch of $-b_{ps}(a^{\text{new}} - a^{\text{old}})t^{\text{new}}$ at the transformer's terminals.

References

- Chan, S. and V. Brandwajn (1986). “Partial matrix refactorization”. In: *IEEE Transactions on Power Systems* 1.1, pp. 193–200 (cit. on pp. [15](#), [16](#), [20](#)).
- Stott, B. and O. Alsac (1974). “Fast decoupled load flow”. In: *IEEE Transactions on Power Apparatus and Systems* 93.3, pp. 859–869 (cit. on pp. [15](#), [16](#)).